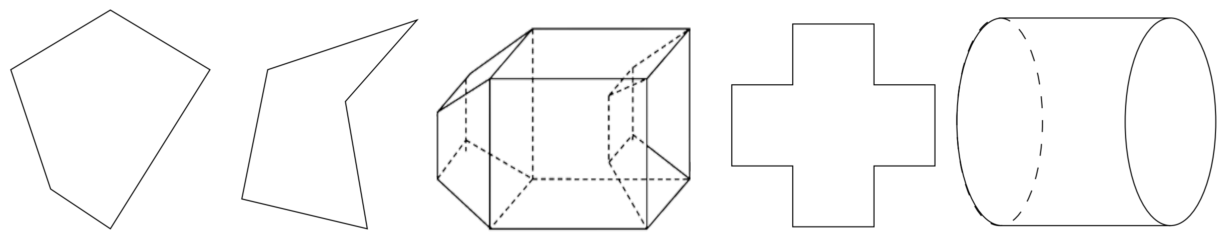
Grade 7/8 Math Circles

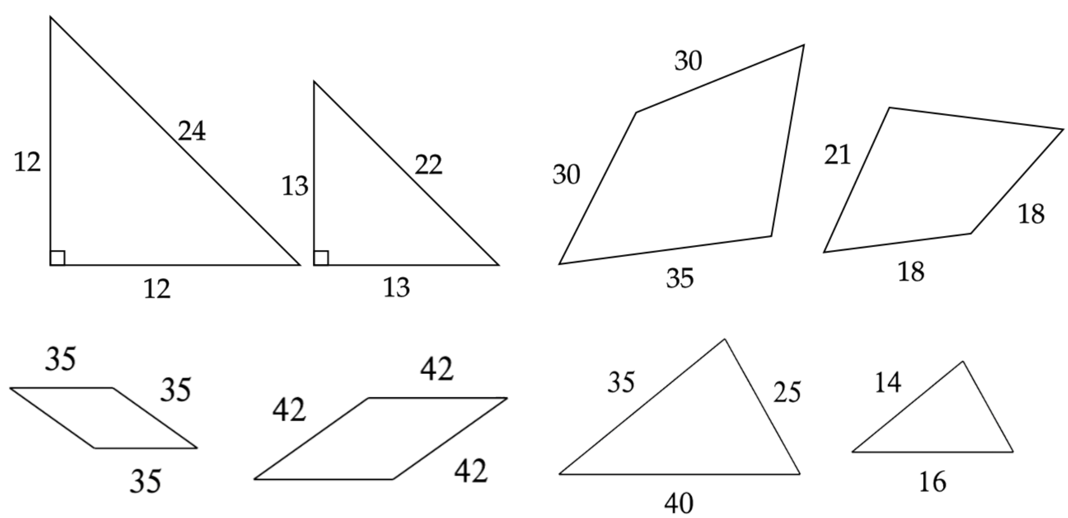
Solids & Platonic Solids

Solids & Platonic Solids - Problem Set

1. What does it mean for a polygon/polyhedron to be convex? What are some methods to check convexity (You should know 3 by now!)?
2. Determine the concavity of the following polygons and polyhedrons (convex or concave).

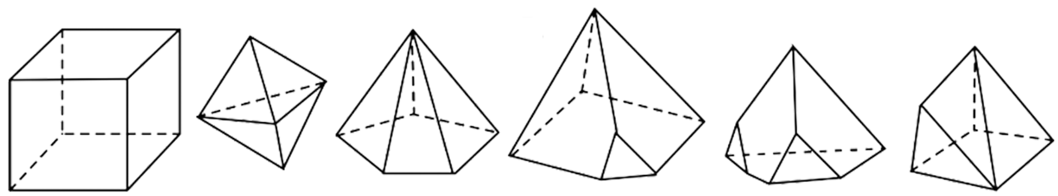


3. Determine the scaling factor of the following similar polygons. Write them as a fraction in lowest terms.



4. Someone tells you that the Schläfli symbol $\{4,5\}$ also represents a Platonic Solid. Are they correct? Why or why not (show mathematically)?

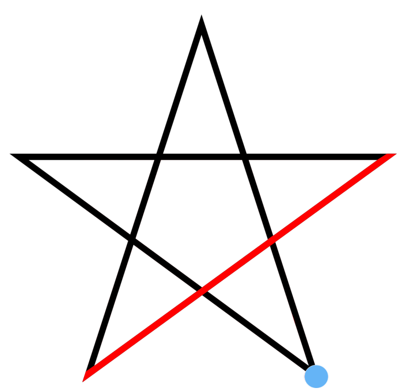
5. Recall Euler's formula is $V - E + F = 2$. Determine the Euler characteristic, χ of the following polyhedra



6. Given the following combination of vertices, edges, and faces (V, E, F), determine the Euler characteristic.

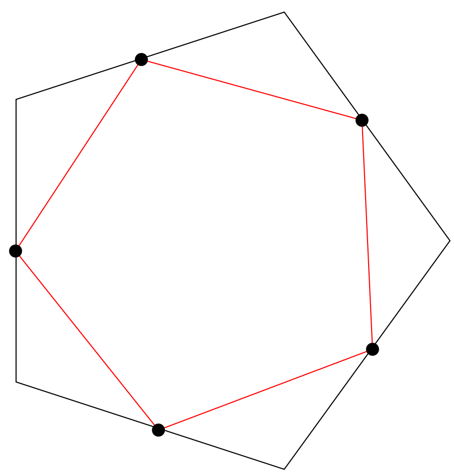
$$(V, E, F) = (6, 12, 7) \qquad (V, E, F) = (12, 24, 12)$$
$$(V, E, F) = (20, 30, 12) \qquad (V, E, F) = (4, 6, 4)$$
$$(V, E, F) = (6, 18, 12) \qquad (V, E, F) = (14, 24, 12)$$

7. You are given a **pentagram** (a star polygon), and are asked to determine its Schläfli symbol. You are told that it's of the form $\{\frac{p}{q}\}$, where where p is the number of vertices and $q - 1$ is the number of vertices skipped when drawing each edge of the star.



   a) Determine the number of vertices, p (note in the diagram, we do not count overlapping lines as a vertex, this is merely for artistic purposes).
   b) Determine the number of vertices skipped (i.e for each red line you connect between two vertices, how many vertices do you skip over). For reference, the blue mark is a skipped vertex. With that, find q.
   c) Write the Schläfli symbol for the pentagram.

8. In geometry, polygons are associated into pairs called duals, where the vertices of one correspond to the edges of the other. This means to construct the dual of a polygon, you first find the midpoint of each edge (the exact middle of an edge), and then connect each of these midpoints in clockwise order. For example, shown below is the dual for the Pentagon. Surprisingly, it's actually another pentagon! (it turns out the duals for any n-gon is itself).



Determine and draw the duals for the following polygons.